## NONSTATIONARY HEAT TRANSFER BASED ON THE SOLUTION OF INVERSE PROBLEMS OF HEAT CONDUCTION USING DATA FROM A FULL-SCALE EXPERIMENT

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The results of an experimental study of nonstationary convective heat transfer between classical bodies in a high-temperature gas flow are presented.

Convective heat transfer in technical installations with time dependent process parameters cannot always be regarded as stationary or quasistationary. The question of whether or not effects of nonstationarity should be taken into account and how requires in a number of practical cases, a careful analysis. From this viewpoint the study of the laws governing nonstationary forced convection on bodies with a very simple geometrical shape, which simplifies the physical interpretation of the results obtained and is a first step in the study of nonstationary heat transfer accompanying flow over complicated structural elements, is of both scientific and practical interest.

Because the solution of conjugate problems of nonstationary heat transfer for most cases of practical importance encounters significant mathematical difficulties (often even in the formulation of the problem), the only way to obtain practical results is by experiment.

In this paper we present the results of a study of nonstationary heat transfer heat transfer accompanying the flow of a high-temperature gas over a circular cylinder and thermally thin plates. The experiments were carried out on a stand for "hot" gas blowing through an array of turbine blades, described in detail in [1].

The objects of the study were (Fig. 1) a hollow cylinder made of 1Khl8N9T steel with an outer diameter of  $6 \cdot 10^{-3}$  m and a wall thickness of  $1 \cdot 10^{-3}$  m, as well as thermally thin platelike junctions of a contact thermal pickup ( $0.26 \cdot 10^{-3}$  m, and  $0.52 \cdot 10^{-3}$  m thick), used for determining the nonstationary temperature of the gas flow by the method of two thermal pickups [2].

The nonstationary thermal state was achieved by rapidly changing the temperature of the gas flow (increasing it or decreasing it) or by a virtually instantaneous (over a time of -0.1 sec) insertion of the object into the gas flow at constant temperature (of the order of 1100 °K).

The nonstationary coefficients of heat transfer from the gas to the hollow cylinder were determined by a procedure based on the solution of a boundary-value [3] inverse problem of heat conduction in a one-dimensional formulation. In so doing the limiting case was realized both in the correct formulation (when in the course of the experiments the temperature of the heat-receiving surface of the heat flux sensor was measured) [4] and in the typically incorrect formulation (when the temperature of the thermally insulated surface of the sensor is measured) [5]. The numerical algorithm was based on the approximation of the one-dimensional heat-conduction equation using an implicit finite difference scheme with iterative inclusion of a nonlinearity of the first kind (temperature dependent  $\lambda$  and c). The temperature measurements  $T_w(x, \tau)$ ;  $T_{f_1}(\tau)$ ;  $T_{f_2}(\tau)$ , serving as the starting data for determining the specific heat flux and the nonstationary temperature of the gas using the method of two thermal pickups, were first smoothed by cubic splines using the method described in [6] in order to increase the accuracy of the solution of the inverse heat-conduction problem and to enable numerical differentiation of the experimental data. The distinguishing characteristics of the construction of the smoothing function minimizing the function [6]

Sergo Ordzhonikidze Ufim Aviation Institute. Translated from Inzhenerno-Fizicheskii Zhurnal, Vol. 49, No. 6, pp. 897-903, December, 1985. Original article submitted May 17, 1985.



Fig. 1. Objects of study: a) circular cylinder: 1) heat flux sensor; 2) F-50 thermocement; 3, 4) thermocouple thermoelectrodes; 5) mica and metallic foil insulation; b) thermally thin plate.

$$\Phi(u) = \int_{a}^{b} [u'']^2 dx + \sum_{k=1}^{n} p_k [u(x_k) - \tilde{f}_k]^2, \qquad (1)$$

were the automatic choice of the weight coefficients  $p_k$  [7] (based on limitation of the maximum admissible relative smoothing error at each nodal point) and automatic evaluation of the amplitude of the relative random error in the starting data (based on the limitation of the number of inflection points in the spline function in a given smoothing interval).

The nonstationary heat-transfer coefficients on the plate-like junctions of the two-junction thermal pickups for determining the temperature of the gas flow were calculated using the formula obtained from the equation for the thermal inertia of contact thermal pickups.

The primary processing of the experimental data and their generalization were carried out on an ES-1033 computer using a program constructed on the basis of the modular principle and written in FORTRAN-4.

The processing of the experimental data for states with stationary heat transfer (when the temperature gradient on the heat-transfer surface of the heat flux sensor  $\partial T_w/\partial \tau$  did not exceed 30°K/sec) enabled establishing that in these states, for transverse flow over a circular cylinder, the experimental results correlate with a high degree of accuracy with the wellknown empirical dependence for calculating stationary heat transfer on the section of the input edge of the blades of turbine machines [8]:

$$Nu_D = 0.635 \, \mathrm{Re}_D^{0.5} \,. \tag{2}$$

It should be noted that the values of the heat transfer coefficients calculated from the indications of the thermal transducers on the heated and thermally insulated surfaces of the heat flux sensor coincided. In analogous states with flow over thermally thin plates (flat junctions of a contact thermal pickup) the experimental data are described to within 20% by the dependence

$$Nu_L = 0.548 \operatorname{Re}_L^{0.5} \operatorname{Pr}_L^{0.33} .$$
(3)

Figure 2 illustrates what was said above.

In analogy to [1], the ratio of the Nu<sub>T</sub> number under nonstationary conditions to the Nu<sub>0</sub> number under stationary conditions with the same values of the state-determining parame-



Fig. 2. Heat transfer in the stationary state: 1)  $Nu_D = 0.635 \text{ Re}_D^{\circ.5}$ ; II)  $Nu_L = 0.548 \text{ Pr}_O^{\circ.5} \text{ Re}_L^{\circ.5}$ . Notation for the cylinder: 1)  $Re_D = (7.6-11.4) \cdot 10^3$ ; 2) (16.0-23.0) \cdot 10^3; 3) (23.0-35.0)  $\cdot 10^3$ ; 4) (7.6-9.0)  $\cdot 10^3$ ; 5) (16.0-18.0)  $\cdot 10^3$ ; 6) (23.0-26.5)  $\cdot 10^3$ ; 7)  $|T_w - T_f|_o = 300^\circ \text{K}$ ; 8) 400; 9) 500; 10)  $T_f = 770$ ; 11) 870; 12) 970. Notation for the plates: 13)  $Re_L = (4.3-4.8) \cdot 10^3$ ; 14) (8.5-9.5)  $\cdot 10^3$ ; 15) (12.6-14.1)  $\cdot 10^3$ ; 16)  $T_f = 970^\circ \text{K}$ ; 17) 1070; 18) 1140; 19)  $h_1 = 0.26 \text{ mm}$ ; 20)  $h_2 = 0.52$ .

ters was chosen to describe quantitatively the effect of the nonstationary temperature on the intensity of heat transfer accompanying flow over the circular cylinder and thermally thin plates

$$\varepsilon_{T\tau} = \frac{\mathrm{Nu}_{\tau}}{\mathrm{Nu}_{0}} , \qquad (4)$$

where  $Nu_0$  is calculated using the dependences (2) and (3) for instantaneous values of the state parameters.

A preliminary analysis of the results obtained showed that a rapid change in the temperature of the gas flow (temperature nonstationarity) for transverse flow over a circular cylinder intensifies heat transfer on the forward part of the cylinder (on an arc of 140°) up to 100-200% (compared with the quasistationary values) and by up to 90% with longitudinal flow over thermally thin plates. The effect of thermal nonstationarity is most strongly manifested at the beginning of the transient process when  $\partial T_w/\partial \tau$  reaches values of 300-1000°K/sec.

The experimental data obtained were generalized using the dimensionless parameter taking into account the effect of the changes in the temperature of the wall on the turbulent structure of the flow and through it on the nonstationary heat transfer [9]:

$$K_{Tg}^{*'} = \frac{\partial T_w}{\partial \tau} \frac{1}{T_w} \sqrt{\frac{\lambda}{c_p g \rho w}}$$
(5)

Application of the methods of the theory of similarity and dimensional analysis to nonstationary convective heat-transfer processes enables obtaining dimensionless determining complexes, consisting of parameters entering into the boundary conditions and appearing as similarity numbers. As correctly pointed out in [10], the canonical homochronicity criterion Ho and Fourier criterion Fo uniquely reflect the effect of nonstationarity only for jump-like perturbations and their monotonic smoothings. It is also pointed out in [10] that nonstationary convective heat-transfer processes are even more individualized than conductive heattransfer processes, and for this reason fixing the boundary conditions in the formulation of boundary-value problems is a complicated problem.

Koshkin et al. [11] were able to find an approximation of a boundary-value function of general form for the temperature of the surface as a function of the coordinates and time by expanding it in a Taylor series around an arbitrary point. Because of the shortness of the length and time intervals in which the effect of the past history of the boundary conditions



Fig. 3. Generalization of the experimental data on the effect of a temperature nonstationarity on heat transfer in a transverse flow over a cylinder (a) and over a plate (b). The notation is the same as in Fig. 2.

is manifested, the boundary value function can with high accuracy be approximated by the first terms of the expansion, including the first derivatives [11]. In those cases when the peculiarities of the nonstationary convective heat transfer process are determined solely by the form of the boundary function (i.e., with the same geometrical and physical conditions of uniqueness), the dimensionless complexes formed from the parameters entering into the boundary function or into its approximation (if such is found) will be the determining similarity numbers. Using different time and length scales, Kalinin et al. [9] and Koshkin et al. [11] obtained a series of modified dimensionless parameters, which together with the starting parameters are also similarity numbers. It should be noted that it is shown in [12] how the same similarity numbers as those presented in [9, 11] can be obtained with the help of classical methods of the theory of similarity based on the system of differential equations for nonstationary forced convective heat transfer (with the corresponding boundary conditions).

Generalization of the experimental data on the study of the effect of the temperature nonstationarity on the intensity of heat transfer between the high-temperature gas flow and a circular cylinder as well as a thermally thin plate confirms the results of the theoretical analysis [9-11] concerning the desirability of using the similarity numbers of the type  $K_{Tg}^{\star^*}$ 

in the similarity equations describing nonstationary forced convective heat transfer. The results of this study were generalized in the form of similarity equations proposed in [11]:  $\epsilon_{T\tau} = 1 + \exp \left[a - b \left| K_{Tg}^{*} \cdot 10^{p} \right|^{-c} \right].$  (6)

The parameters of the averaged empirical dependences (Fig. 3, 4) described by Eq. (6), as well as the regions of variation of the numbers of Re and the complex  $K_{Tg}^{*1}$  studied are presented in Table 1. Here the diameter of the circular cylinder and the length L of the rectangle equal to the segment indicated in Fig. 1 were chosen as the characteristic dimensions in determining the numbers  $Re_D$  and  $Re_L$ . The instantaneous value of the gas temperature  $T_f(\tau)$  was taken as the determining temperature.

Object of study	State	Parameters				Range of applicability	
		а	Ь	c	р	ReD; ReL	$K_{Tg}^{*} \cdot 10^{5}$
Circular cylinder	$Emission T_f$	3,8	4,84	0,368	5	$Re_D = (7,6 - 36,5) \cdot 10^3$	-(0,1-3,1)
	Heating:	ī,1	3,57	0,286	5	$\begin{array}{c} \operatorname{Re}_{D} = (7,6 - \\ 35,1) \cdot 10^{3} \end{array}$	0,1— 7,75
Thermally thin plate	$T_f$	-0,08	0,074	1,94	4	$\begin{array}{c} \text{Re}_{L} = (4,3 - 14,1) \cdot 10^{3} \end{array}$	- 130

## TABLE 1. Parameters and Ranges of Applicability of the Generalizing Dependences (6)

## NOTATION

c, heat capacity; w, velocity; T, temperature; g, acceleration of gravity; Nu, Re, and Pr, Nusselt, Reynolds, and Prandtl numbers, respectively;  $K_{Tg}^{**}$ , dimensionless criterion of

temperature nonstationarity;  $\lambda$ , coefficient of thermal conductivity;  $\rho$ , density; and  $\tau$ , instataneous time. Indices: w, wall; f, flow; L, plate; D, cylinder.

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